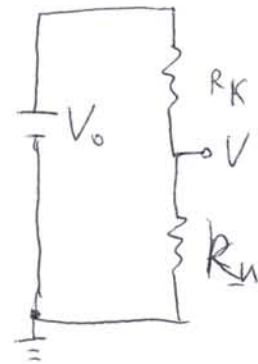


Resistor divider error analysis:

$$V = V_0 \frac{R_u}{R_u + R_k}$$

$$V R_u + V R_k = V_0 R_u$$

$$(V_0 - V) R_u = V R_k$$



$$R_u = \frac{V}{V_0 - V} R_k$$

$$dR_u = R_k \frac{d}{dV} \left( \frac{V}{V_0 - V} \right) = \left[ (dV) \frac{1}{V_0 - V} + V (+1) \frac{1}{(V_0 - V)^2} dV \right] R_k$$

$$= \frac{R_k dV}{(V_0 - V)^2} (V + V_0 - V) = \frac{R_k V_0}{(V_0 - V)^2} dV$$

$$\frac{dR_u}{R_u} = \frac{V_0 R_k}{(V_0 - V)^2} \frac{V_0 - V}{V R_k} dV = \frac{V_0 dV}{V(V_0 - V)} = \frac{dV}{V} \left( \frac{V_0}{V_0 - V} \right) = \frac{dV}{V} \left( \frac{V_0}{V_0 \left( 1 - \frac{R_u}{R_u + R_k} \right)} \right)$$

$$= \frac{dV}{V} \left( \frac{R_u + R_k}{R_k} \right) = dV \frac{(R_u + R_k)^2 R_u + R_k}{V_0 R_k R_u V_0 R_u}$$

$$RE = \frac{dR_u}{R_u} = \frac{(R_u + R_k)^2}{R_u R_k} \left( \frac{dV}{V_0} \right)$$

$$\frac{dV}{V_0} = \frac{1}{1024}$$

$$RE = \frac{1}{1024} \frac{4 R_{avg}^2}{R_{geo}^2}$$

$$R_{avg} = (R_u + R_k) / 2$$

$$R_{geo} = \sqrt{R_u \cdot R_k}$$